Parallel or Perpendicular

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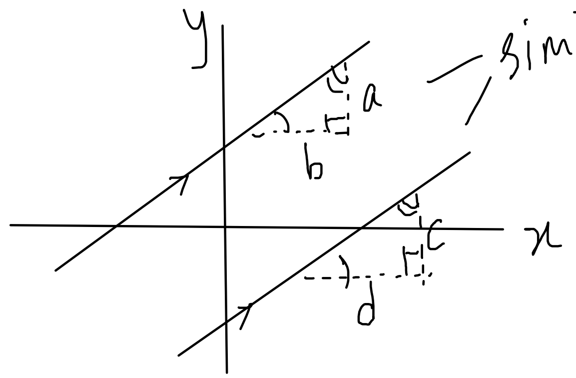
Parallel lines



similar Δs

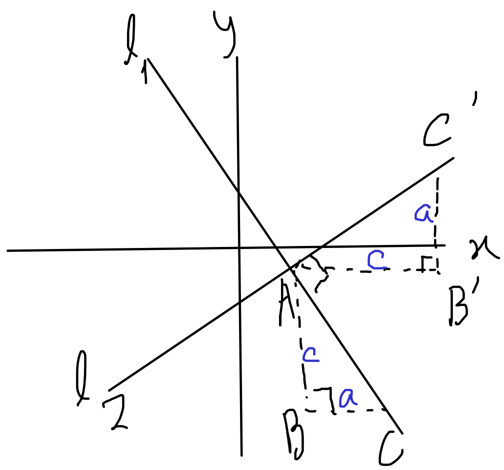
$$\frac{a}{b} = \frac{c}{d}$$

$\therefore$ same gradients

e.g. gradient

$$\left.\begin{aligned} y &= 2x+3 \\ y &= 2x-2 \end{aligned}\right\} \text{ are parallel}$$

Perpendicular Lines.



Can rotate ΔABC to $\Delta AB'C'$.

$$\left.\begin{aligned} &\text{Gradient of } l_1 \text{ is} \quad m_1 = -\frac{c}{a} \\ &\quad " \quad \text{of } l_2 \quad " \quad m_2 = \frac{a}{c} \end{aligned}\right\} \therefore m_1 m_2 = -1.$$

e.g.

$$\left.\begin{aligned} y &= 2x+3 \\ y &= -\frac{x}{2} - 1 \end{aligned}\right\} \text{ are perpendicular}$$

Midpoint

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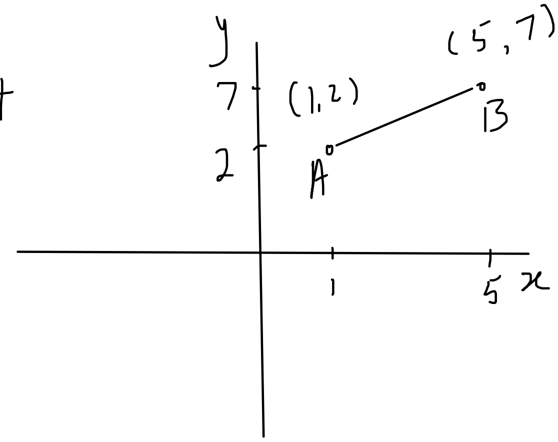
e.g. Find the midpoint of AB.

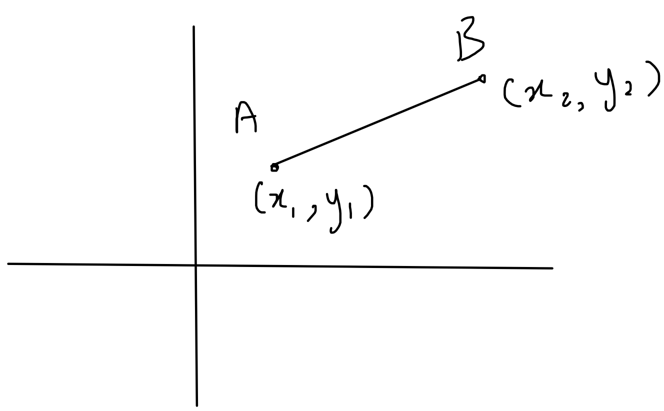


Take mean of x values : $\frac{1+5}{2} = 3$

and " " y " : $\frac{2+7}{2} = \frac{9}{2}$

Midpoint : $\left(3, \frac{9}{2}\right)$

Formula



Midpoint of AB is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Graphs of parabolas with equations in the form $y^2 = kx$

$y^2 = kx$

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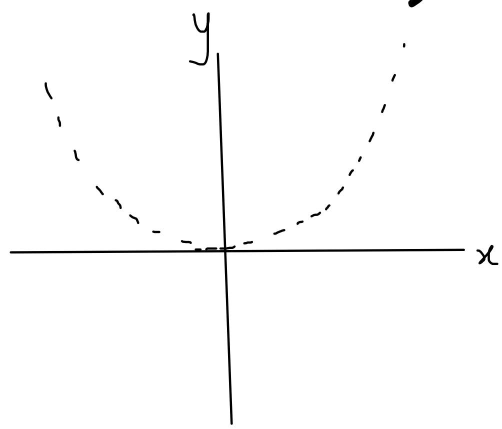
e.g. $y = x^2$

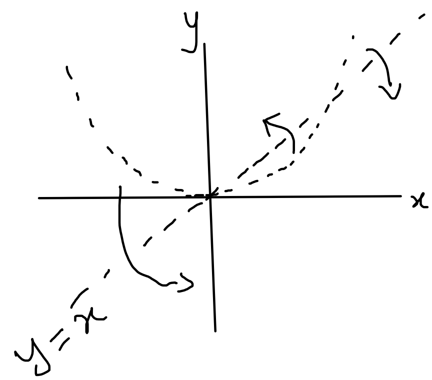


e.g. $y^2 = x$

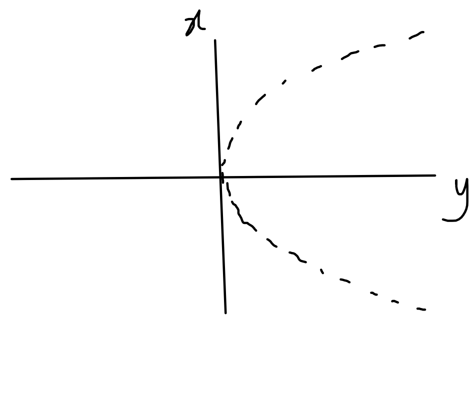
To sketch, exchange x and y axes,

1. Take graph above

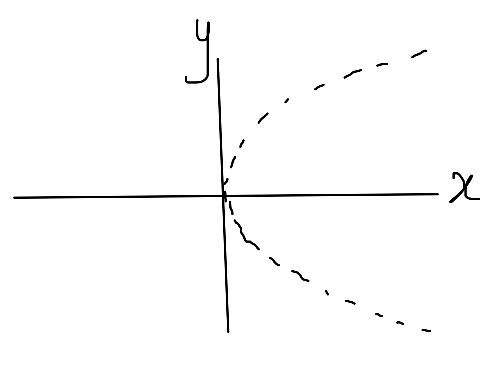
Flip about $y = x$



2. Get this



3. Rename x to y, and y to x



$y^2 = x$

Coordinate geometry of circles in the form $(x-a)^2 + (y-b)^2 = r^2$

Circles 1

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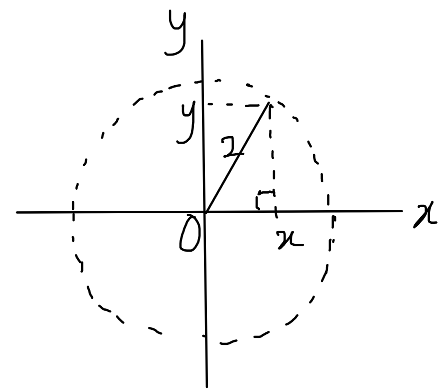
e.g. Find formula for a circle graph of radius 2 with centre at O.

ans. Pythagoras theorem:

$$x^2 + y^2 = 2^2$$



True for any point (x, y) on circle,

$\therefore$ equation of circle graph is $x^2 + y^2 = 2^2$

For circle of radius r centred at O,

equation is $\boxed{x^2 + y^2 = r^2}$

Coordinate geometry of circles in the form $x^2+y^2+2gx+2fy+c=0$

Circles 2

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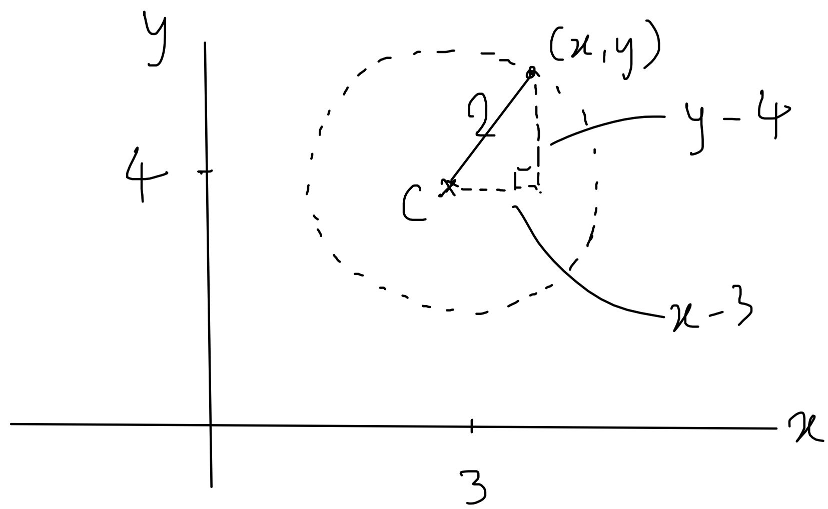
e.g. Find formula for a circle graph of radius 2 with centre at $(4,3)$.



C is centre.
(x,y) is on circle.

Pythagoras:

$$(x-3)^2+(y-4)^2=2^2$$

For circle of radius r centred at (a,b)

equation is $\boxed{(x-a)^2+(y-b)^2=r^2}$ (I)

Alternate form - expand:

$$x^2-2ax+a^2+y^2-2by+b^2=r^2$$

$$x^2+y^2+(-2a)x+(-2b)y+(a^2+b^2-r^2)=0$$

Write as $\boxed{x^2+y^2+2gx+2fy+c=0}$ (II)

e.g. Find the radius and centre of the circle $x^2+y^2-6x-8y+21=0$.

Ans. 1. Complete the squares for x^2-6x and y^2-8y separately. 2. Then arrange into form (I).

Transformation of given relationships, including $y = ax^n$ and $y = kb^x$, to linear form to determine the unknown constants from a straight line graph.

Transform to Linear Form

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e.g. Given a table of values

x	1	2	3	4	5
y	0.9	4.1	8.8	16.2	24.7

Given $y = ax^n$ but a, n unknown.
Find a, n.

Ans. Take logarithm:

$$\log_{10} y = \log_{10} a + \log_{10} x^n$$
$$\log_{10} y = n \log_{10} x + \log_{10} a$$

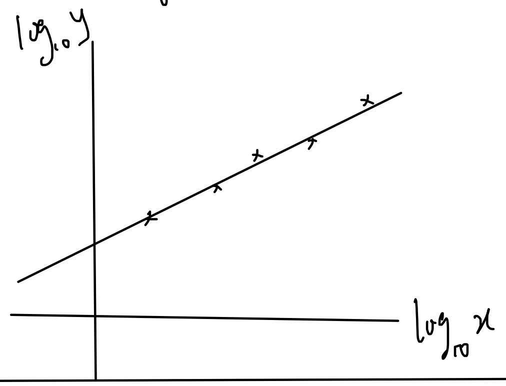
n, $\log_{10} a$ unknowns.
Can find by substituting 2 sets of (x, y) from table and solving the 2 simultaneous equations.

But if table comes from some measured values, values have some errors.

Better way - plot $\log_{10} y$ against $\log_{10} x$.

A straight line:

$$\underline{\log_{10} y} = \underline{n} \, \underline{\log_{10} x} + \underline{\log_{10} a}$$
$$Y = mX + c$$



So n = gradient
$\log_{10} a$ = y-intercept.

If $y = kb^x$ instead, can do

$$\log_{10} y = \underbrace{(\log_{10} b)}_{\text{gradient}} x + \underbrace{\log_{10} k}_{\text{intercept}}$$
$$Y = mX + c$$

- plot $\log_{10} y$ against x.

Problem 1

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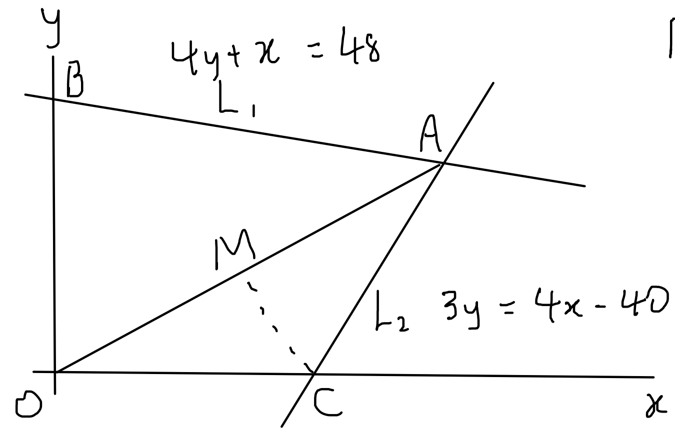
2013 P1 Q10



M is the midpoint of OA.

(i) Show that $\angle OMC = 90^\circ$

(ii) Find ratio of areas $\Delta OAB : \Delta OAC$.

Solution.

(i) $C:$ $y = 0,$ $L_2:$ $0 = 4x - 40$

$x = 10.$

$A:$ L_1 $x + 4y = 48$ — (1)

L_2 $4x - 3y = 40$ — (2)

$4 \times (1):$ $4x + 16y = 192$ —(3)

$(3) - (2):$ $19y = 152$

$y = 8$

$\rightarrow (1):$ $x + 4(8) = 48$

$x = 16$ $\Rightarrow$ $A: (16, 8)$

$M:$ $\left(\frac{16}{2}, \frac{8}{2}\right) = (8, 4)$

Gradient of OM $= \frac{4}{8} = \frac{1}{2}$

$C: (10, 0)$

Gradient of CM $= \frac{0-4}{10-8} = -2$

product $= -1$

$\therefore \angle OMC = 90^\circ$

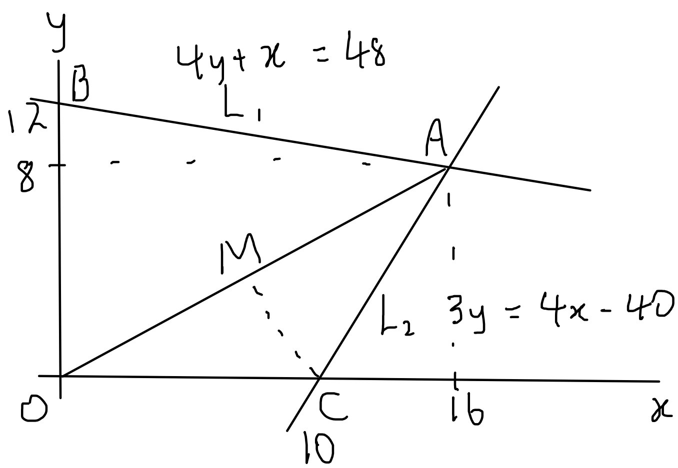
Problem 1

Dr. K. M. Hock

(ii) B: $x = 0$, L_1: $4y + 0 = 48$

$$y = 12$$

B: $(0, 12)$, A: $(16, 8)$, C: $(10, 0)$



$$\Delta OAB = \frac{1}{2} \times 12 \times 16$$

$$\Delta OAC = \frac{1}{2} \times 10 \times 8$$

$$\frac{\Delta OAB}{\Delta OAC} = \frac{12 \times \cancel{16}^{2}}{\cancel{10}_{5} \times \cancel{8}}$$

$$\Delta OAB : \Delta OAC = 12 : 5$$

Problem 2

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2013 P1 Q13 The value $V of a diamond is related to t, the number of years since it was mined in 1970. The variables V and t are related by the formula $V = ae^{kt}$, where a and k are constants. The table above gives the value of the diamond in 1980, 1990, 2000 and 2010.

Year	1980	1990	2000	2010
t (years)	10	20	30	40
V($)	8000	9200	10600	12200

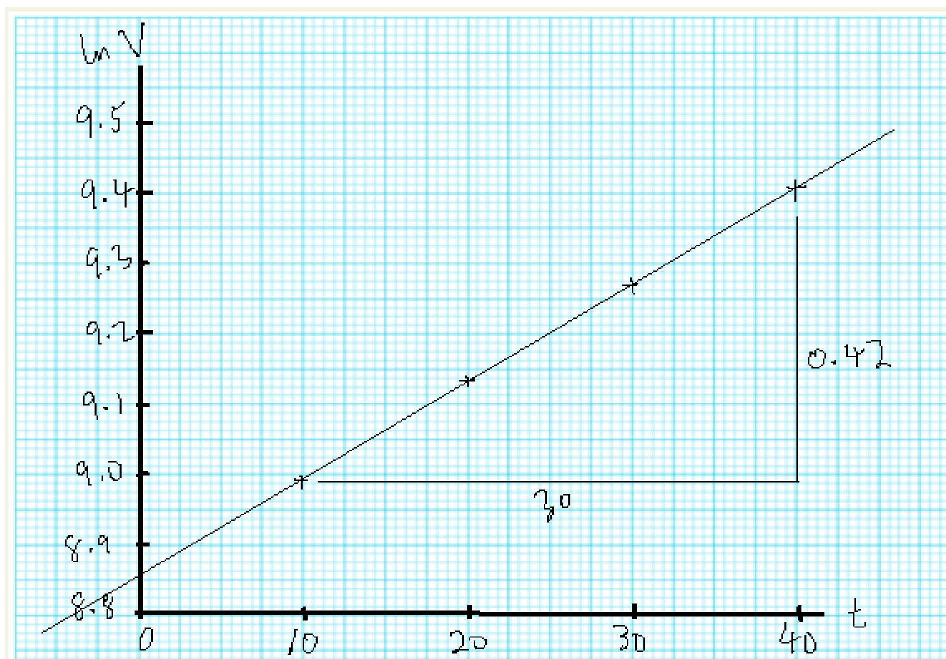
(i) On graph paper, plot ln V against t and draw a straight line graph. The vertical ln V-axis should start at 8.8 and have a scale of 2 cm to 0.1.

(ii) Use the graph to estimate the value of a and of k.

(iii) Estimate the value of the diamond in 2014.

Solution (i) $\ln V = \ln a + kt$

t	10	20	30	40
ln V	8.99	9.13	9.27	9.41



(ii) $\ln a = \text{y-intercept}$

$= 8.86$

$a = 7044$

$k = \text{gradient}$

$= \frac{0.42}{30} = 0.014$

(iii) $t = 44$

$V = 7044e^{0.014t}$

$= 7044e^{0.014 \times 44} = 13040$

Problem 3

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2013P2Q10 A circle has the equation $x^2 + y^2 + 6x - 4y - 12 = 0$.

(i) Find the coordinates of the centre of the circle and the radius of the circle.

(ii) Show that the equation of the tangent to the circle at the point $P(-7, -1)$ is $3y + 4x + 31 = 0$.

The point Q, which lies on the circle, is the same distance from the y-axis as the point P.

(iii) Find the equation of the tangent to the circle at Q.

The tangents to the circle at P and Q intersect at the point R.

(iv) Find the coordinates of R.

Solution

(i)

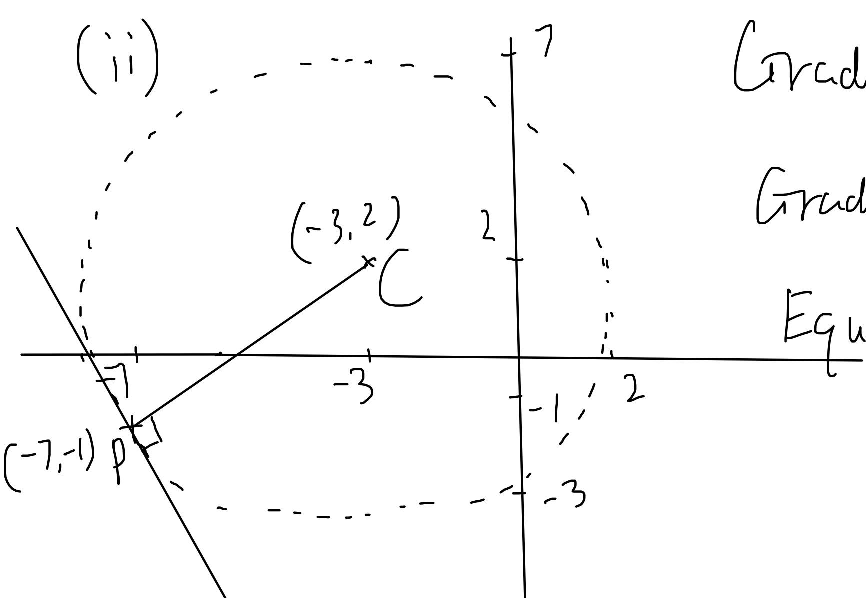
$$x^2 + 6x + y^2 - 4y - 12 = 0$$

$$(x+3)^2 - 3^2 + (y-2)^2 - 2^2 - 12 = 0$$

$$(x+3)^2 + (y-2)^2 = 9 + 4 + 12 = 25$$

Centre : $(-3, 2)$ radius $= 5$

(ii)



Gradient of CA $= \dfrac{2-(-1)}{-3-(-7)} = \dfrac{3}{4}$

Gradient of line $= -\dfrac{4}{3}$

Equation: $-\dfrac{4}{3} = \dfrac{y-(-1)}{x-(-7)}$

$$-\frac{4}{3} = \frac{y+1}{x+7}$$

$$-4x - 28 = 3y + 3$$

$$0 = 3y + 4x + 31$$

Problem 3

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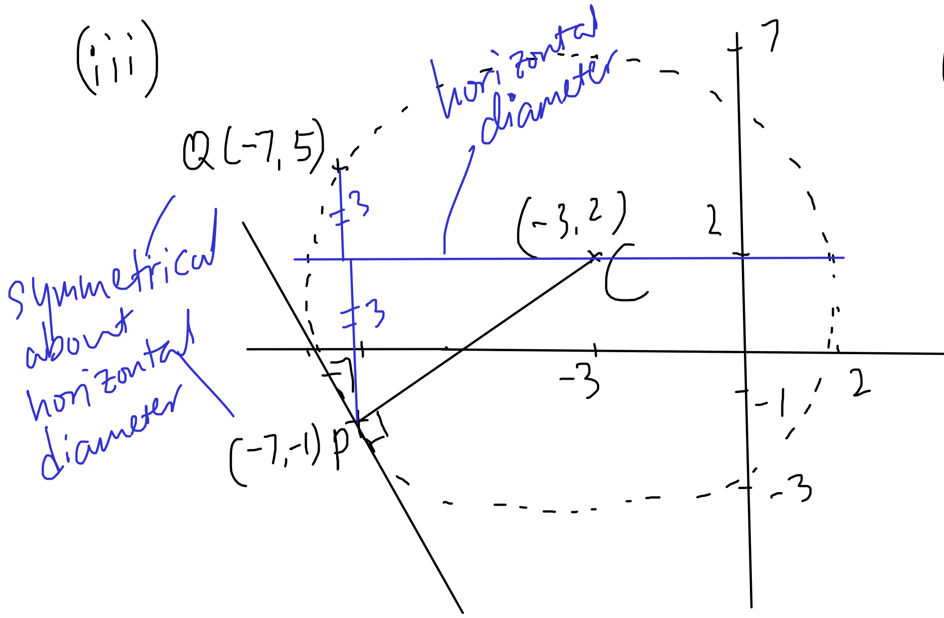
(iii)



Q same distance above C as P is below C.

Tangent at Q has same slope a tangent at P, but positive sign.

$\rightarrow \; +\frac{4}{3}.$

Equation: $+\frac{4}{3} = \frac{y-5}{x-(-7)}$

$$4x + 28 = 3y - 15$$

(I) — $0 = 3y - 4x - 43$

(iv) Tangents at P and Q symmetrical about horizontal line thru C.

$\therefore$ y-value of R = 2.

(I) $\rightarrow$ $0 = 3(2) - 4x - 43$

$$4x = -37$$

$$x = -\frac{37}{4}$$

R : $\left(-\frac{37}{4}, 2\right)$